TM No. 841097

NAVAL UNDERWATER SYSTEMS CENTER
NEW LONDON LABORATORY
NEW LONDON, CT 06320

Technical Memorandum

THE EFFECT OF HYDROPHONE CONFIGURATION ON THE DIRECTIVITY INDEX OF A LINE ARRAY

Date: 11 June 1984

Prepared by: Peter D. Herstein
Peter D. Herstein
Physicist, 33A3

DEPARTMENT OF THE NAVY
NAVAL UNDERWATER SYSTEMS CENTER

NEWPORT LABORATORY
NEWPORT, RI 02841

NEW LONDON LABORATORY
NEW LONDON, CT 06320

IN REPLY REFER TO:

33A3:PDH:alb
5600
Ser: 433A3-114

JUL 16 1984

From: Commanding Officer
To: Commander, Naval Sea Systems Command, Naval Sea Systems Command Headquarters (Attn: C. D. Smith, Code 63R), Washington, DC 20362

Subj: NUSC Technical Memorandum; forwarding of

Encl: (1) Peter D. Herstein, "The Effect of Hydrophone Configuration on the Directivity Index of A Line Array," NUSC TM 841097, 11 June 1984

1. One task of the Broadband Passive Sonar Signal Processing Program is in the area of Large Array Bearing-Time processing. As a part of this task, a study has been conducted investigating the effect of hydrophone configuration on the directivity index of a line array. Three array configurations were examined: asymmetric geometric taper, symmetric geometric taper, and equi-spaced. The Directivity index for each array was computed as function of both frequency and steering angle. The DI values for the symmetric and asymmetric arrays are very similar, and show less fluctuation over frequency and steering angle than the DI values for the equi-spaced array configuration. For all three arrays, when the receiving frequency is less than the designed minimum frequency, the maximum value of DI as a function of steering angle is at endfire (0°).

2. Enclosure (1), documenting the results of this study, is forwarded for your interest and retention.

3. Any questions concerning this enclosure should be directed to Mr. Peter D. Herstein, Code 33A3, at (203)440-4970.

T. E. Bateman
L. FREEMAN
By direction

33A3:PDH:alb
5600
Ser: 433A3-114

Copy to: (w/encl)

DARPA	(CDR K. Evans)
CNO	(OP-095; -098; CAPT E. Young, CDR H. Dantzler, 952D) (4)
CNM	(SPO PM-2; MAT-0731 (T. Kooij), -0724 (CAPT J. Harlett), PM-4 (J. Sinsky) (4)
NAVELEX	(R. Mitnick, J. Schuster, T. Higbee, 612) (3)
NAVSEASYSCOM	(SEA-63R; D. Porter, J. Shooter, 63R; P. Talbert, L. Kneesi, 63D) (5)
NAVPGSCOL	(1)
NORDA	(R. Lauer, 320; R. Martin, 150; D. Ramsdale, 345; G. Stanford, 115T, W. Worsley, 530; Library) (6)
NADC	(J. Howard, B. Steinberg, J. Gombali; M. Higgins, 3032; Library) (5)
NSWC	(R. Stevenson, M. Stripling, E. Hein, T. Harris; Library) (5)
NRL	(R. Dicus, R. Doolittle, 5160; Library) (3)
MPL	(F. Fisher, B. Williams) (2)
DTIC	(2)
DIA	(1)
ARL/UT	(N. Bedford, P. Vidmar) (2)
NOO	(W. Jobst, J. Allen, 7310; R. Merrifield, 7311; R. Swenson, 6200; R. Christensen) (5)
NOSC	(L. Arndt, D. Barbour, R. Bolam, C. Persons, 7133; R. Albrecht, J. McCarthy, 7134; M. Pederson, 724; R. Harris, 7231; Library) (9)
NAVAIR	(370B (E. Davis), 370J (W. Parigian)) (2)

Internal: (w/o encl)

33	
33A	(B. Cole, S. Santaniello)
33A1	(G. Mayer)
33A3	(P. Herstein)
3322	(W. Birtcher)

P. D. HERSTEIN
X4970
12 JUNE 1984

TM No. 841097

NAVAL UNDERWATER SYSTEMS CENTER
NEW LONDON LABORATORY
NEW LONDON, CT 06320

Technical Memorandum

THE EFFECT OF HYDROPHONE CONFIGURATION ON THE DIRECTIVITY INDEX OF A LINE ARRAY

Date: 11 June 1984

Prepared by: Peter D. Herstein
Peter D. Herstein
Physicist, 33A3

ABSTRACT

The directivity index (DI) is a special case of array gain for the constraint that the signal is a plane wave and the noise field is isotropic. To examine the effect of hydrophone configuration on DI, three arrays have been examined. All three arrays are line arrays nominally 300 m in length and composed of 32 omnidirectional elements. The three array configurations are asymmetric geometric taper, symmetric geometric taper, and equi-spaced. Directivity index values were computed over the frequency regime of 10 to 410 Hz, and over the steering angle regime of broadside (90°) to endfire (0°). The DI values for the asymmetric and symmetric are very similar, and show less fluctuations over frequency and steering angle than the DI values for the equi-spaced array configuration. For each of the three arrays, at all frequencies below a cutoff frequency which depends on the array configuration type, the maximum value of DI as a function of steering angle is at endfire (0°).

ADMINISTRATIVE INFORMATION

This work was performed under the Broadband Passive Sonar Signal Processing program, Program Manager Mr. D. Porter, SEA 63R. Technical Director of this special focus technology program is Dr. C. Persons, NOSC Code 7133. Principal Investigator for the Large Array Bearing-Time Processing task is Mr. G. Mayer, NUSC Code 33A1. The work is funded under program element 62711N, sub project SF11123001. The author of this report is located at the Naval Underwater Systems Center, New London Laboratory, New London, CT 06320.

ACKNOWLEDGMENTS

The author would like to thank Mr. William Birtcher and Mr. Gerald Mayer for their comments and suggestions, and Ms. Christine Perry for her programming assistance.

TABLE OF CONTENTS

	Page
ABSTRACT	i
ADMINISTRATIVE INFORMATION	i
ACKNOWLEDGEMENTS	i
LIST OF FIGURES	iv
INTRODUCTION	1
BRIEF MATHEMATICS	1
ARRAY CONFIGURATIONS	1
COMPUTATIONAL METHOD	3
RESULTS	4
FIGURES	6
REFERENCES	17

LIST OF FIGURES

Figure	Title	Page
1	Array Coordinate System for DI Study	6
2	Generalized Geometric Taper Array Design	7
3	Universal Beampattern of the Asymmetric Array	8
4	DI vs Frequency of the Asymmetric Array for Four Steering Angles	9
5	Contours of DI for the Asymmetric Array	10
6	Universal Beam Pattern of the Symmetric Array	11
7	DI vs Frequency of the Symmetric Array for Four Steering Angles	12
8	Contours of DI for the Symmetric Array	13
9	Universal Beam Pattern of the Equi-Spaced Array	14
10	DI vs Frequency of the Equi-Spaced Array for Four Steering Angles	15
11	Contours of DI for the Equi-Spaced Array	16

THE EFFECT OF HYDROPHONE CONFIGURATION ON THE DIRECTIVITY INDEX OF A LINE ARRAY

INTRODUCTION

The directivity index, or DI, is a special case of array gain for the condition when the signal is a plane wave and the noise field is isotropic. It is a frequently used parameter for the assessment of an array's utility. As a part of the Broadband Passive Sonar Signal Processing program, the directivity indices for three arrays have been investigated as functions of both frequency and steering angle. All three arrays each have 32 hydrophones and a line aperture on the order of 300 meters, but each array has different hydrophone spacings. The three hydrophone spacings are (1) asymmetric geometric taper (2) symmetric geometric taper, and (3) equi-spaced. This memorandum describes the principal results of the DI study.

BRIEF MATHEMATICS

Consider a line array with a Cartesian co-ordinate system as shown in figure 1. The pressure squared far-field beam pattern can be defined as $b(f,\theta,\theta_s,\phi)$, where f is frequency, θ is the bearing angle in the x-y plane, θ_s is the steering angle in the x-y plane, and ϕ is the bearing angle in the x-z plane. In this study the beam pattern is assumed to have rotational symmetry about the x-axis. The beam pattern can then be defined without ambiguity as $b(f, \theta, \theta_s)$, and the DI can be defined as

$$DI\ (f, \theta_s) = -10 \log_{10}((1/2)\int_{0^\circ}^{180^\circ} b(f, \theta, \theta_s) \sin\theta \, d\theta) \tag{1}$$

ARRAY CONFIGURATIONS

Three hydrophone configurations were considered. All three are line arrays consisting of 32 elements. All were developed from the generalized geometric taper design,[2] as shown in figure 2, where

S_{min} = minimum spacing between hydrophones

S_{max} = maximum spacing between hydrophones

N = total number of hydrophones

$$r = R^{1/(N-2)} \tag{2}$$

$$R = S_{max}/S_{min} = \text{taper ratio} \tag{3}$$

If the sound speed is defined as C, two other useful definitions are

$$f_{max} = C/(2S_{min}) = \text{maximum design frequency} \tag{4}$$

$$f_{min} = C/(2S_{max}) = \text{minimum design frequency} \tag{5}$$

R can now be redefined as $R = f_{max}/f_{min}$. The three array designs considered are asymmetric, symmetric, and equi-spaced. The principal properties of the three designs are given in Table A, below -

Table A - Properties of Three Proposed Line Arrays

Array No.	Hydrophone Configuration	f_{min} (Hz)	f_{max} (Hz)	S_{max} (m)	S_{min} (m)	L (m)	R	N
1	Asymmetric	49	197	15.2	3.8	295.3	4	32
2	Symmetric	49	197	15.2	3.8	301.0	4	16
3	Equi-spaced	75.0	75.0	10.0	10.0	310.0	1	32

For the above table and throughout this study, the sound speed C is assumed to be 1500 m/s.

Array Number 1, the asymmetric geometric configuration, was obtained directly from the generalized geometric taper design. Array Number 2, the symmetric geometric configuration, was obtained by joining two asymmetric arrays each consisting of 16 elements. This design is called 'symmetric' because the element spacings are minimum at the center of the array, and increase symmetrically toward both ends of the array. Array Number 3, the equi-spaced array, is a special case of the asymmetric geometric taper, i.e., the taper ratio is 1. The element location for all three configurations are given in Table B, as follows:

Table B - Element Locations for the Three Proposed Arrays

Element No.	Element Location (m) Asymmetric	Symmetric	Equi-spaced
1	0.0	0.0	0.0
2	3.8	15.2	10.0
3	8.0	29.7	20.0
4	12.6	43.4	30.0
5	17.5	56.4	40.0
6	22.9	68.6	50.0
7	28.9	80.1	60.0
8	34.7	90.7	70.0
9	41.2	100.6	80.0
10	48.0	109.7	90.0
11	55.2	118.1	100.0
12	62.9	125.7	110.0
13	70.9	132.6	120.0
14	79.3	138.7	130.0
15	88.0	144.0	140.0
16	97.2	148.6	150.0
17	106.7	152.4	160.0
18	116.6	157.0	170.0
19	126.9	162.3	180.0
20	137.5	168.4	190.0
21	148.6	175.3	200.0
22	160.0	182.9	210.0
23	171.8	191.3	220.0
24	184.0	200.4	230.0
25	196.6	210.3	240.0
26	209.6	221.0	250.0
27	222.9	232.4	260.0
28	236.6	244.6	270.0
29	250.7	257.6	280.0
30	265.2	271.3	290.0
31	280.0	285.8	300.0
32	295.3	301.0	310.0

COMPUTATIONAL METHOD

The universal beam pattern was computed for each array configuration. The universal beam pattern $b(\gamma)$ is the pressure squared far-field response of an array as a function of γ, which is defined as

$$\gamma = (2\ f\ S_{min}/C)(\cos\theta - \cos\theta_s) \tag{6}$$

The DI of each particular frequency/steering angle pair was computed numerically by integrating eq. 1 over the appropriate bounds of the universal beam pattern. For example, consider the DI computation for the symmetric

configuration at a frequency of 100 Hz and steering angle of 30°. The conventional beampattern would be integrated from 0° to 180°. The universal beampattern is integrated from γ_{min} to γ_{max}, where

$$\gamma_{min} = (2 \times 100 \times 3.8/1500)(\cos(180^\circ) - \cos(30^\circ))$$
$$\gamma_{min} = -.95$$
$$\gamma_{max} = (2 \times 100 \times 3.8/1500)(\cos(0^\circ) - \cos(30^\circ))$$
$$\gamma_{max} = +.07$$

The advantage of using the universal beam pattern instead of the conventional beam pattern is that the universal beam pattern need only be computed once for each array configuration, while the conventional beam pattern must be computed individually for each frequency/steering angle required.

The software was tested by comparing results with analytic DI solutions for an N element equi-spaced array steered broadside.[1] For N = 32, and a separation of 10m between elements, DI's were computed from 10.0 Hz to 410.0 Hz in 1.0 Hz increments. The maximum error was 0.01 dB.

RESULTS

Figure 3 shows the universal beam pattern for the asymmetric array over the range of γ from -2.0 to 2.0. Figure 4 shows the DI of the asymmetric array as a function of frequency from 10 to 410 Hz for four values of steering angle. The solid line shows results obtained with a steering angle of 90° (broadside), the dashed line represents results for 60° steering, the plus symbols represent results for 30° steering, and the dots represent results at 0° (endfire) steering. The arrow on the y axis, at 15 dB, represents $10 \log_{10} 32$. In general, at any given frequency, values of DI at $\theta_s = 60^\circ$ are less than those at $\theta_s = 90^\circ$, and values of DI at $\theta_s = 30^\circ$ are less than those at $\theta_s = 60^\circ$. However, note the results at endfire steering ($\theta_s = 0^\circ$). For frequencies less than 150 Hz, DI values at endfire are actually greater than those at broadside. To further understand the behavior of the DI as a function of frequency and steering angle, figure 5 is a contour plot of the DI of the asymmetric array. The region in white represents all values of DI greater than 15 dB, while the region in black represents values of DI between 12 and 15 dB, and the hatched area represents all values less than 12 dB.

Figure 6 shows the universal beampattern of the symmetric array, plotted from $\gamma_{min} = -2.0$ to $\gamma_{max} = 2.0$. Figure 7 shows the DI of the symmetric array as a function of frequency for four steering angles. The solid, dashed, plus symbol, and dotted lines show results for steering angles of 90°, 60°, 30°, and 0°, respectively. Again, the arrow marks a level of 10 $\log_{10}32$, or 15 dB. In general, results are very similar to those of the asymmetric array. Figure 8 shows the contour plot of the symmetric array, and again is quite similar to figure 5.

Figure 9 shows the universal beampattern of the equi-spaced array, plotted from $\gamma_{min} = -2.0$ to $\gamma_{max} = 2.0$. Figure 10 shows the DI of the equi-spaced array as a function of frequency for the same four steering angles used for the other two array configurations. Again, the solid, dashed, plus symbol, and dotted lines show DI results for steering angles of 90°, 60°, 30° and 0°, respectively. The arrow marks 10 $\log_{10}32$, or 15 dB. For broadside steering, note the maximum of 17.5 dB obtained at 150 Hz, which suddenly drops off by about 4 dB within just a few Hertz. Figure 11 shows the associated contour plot, which is strikingly different from the contour plots of the asymmetric and symmetric arrays.

SUMMARY

The directivity indices for three array configurations were examined as function of frequency and steering angle. All three arrays are line arrays nominally 300 m in length and consisting of 32 elements. The three configurations are asymmetric geometric taper symmetric geometric taper, and equi-spaced. The DI performance of the asymmetric and symmetric arrays are very similar, and show less fluctuations over frequency and steering angle than DI values for the equi-spaced array. For each of the three arrays, at all frequencies below a cutoff frequency which depends on the array configuration type the maximum value of DI as a function of steering array is at endire (0°).

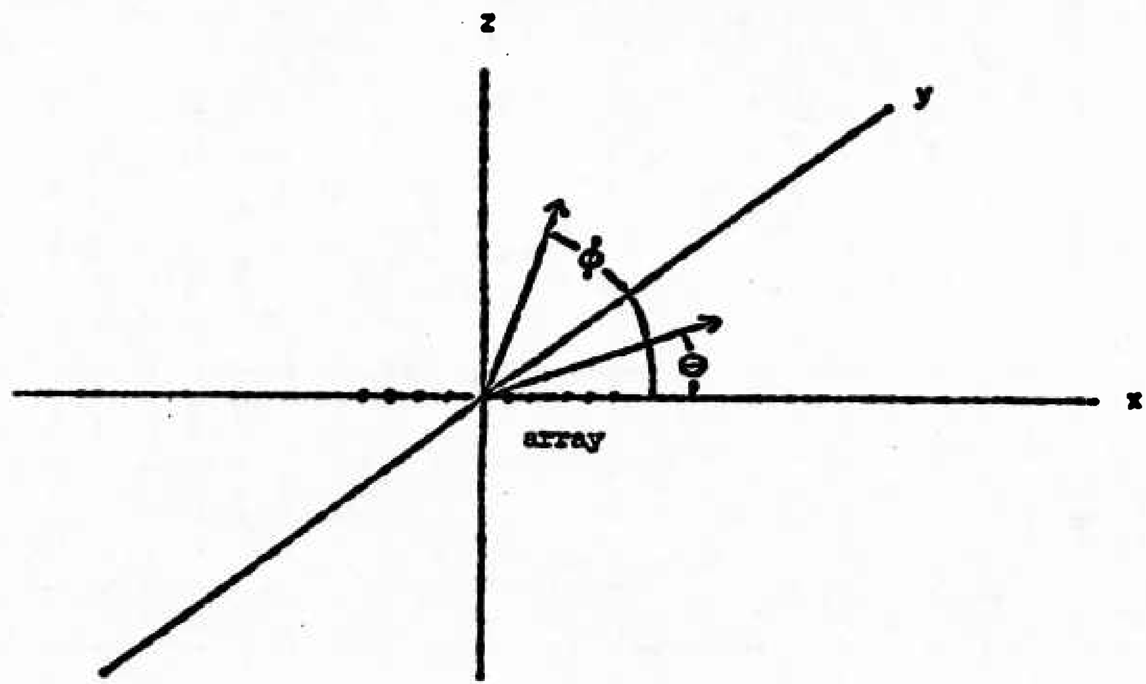

Figure 1. Array Coordinate System for DI Study

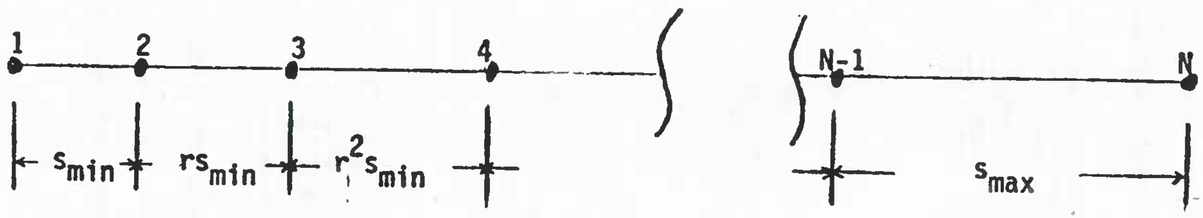

Figure 2. Generalized Geometric Taper Array Design

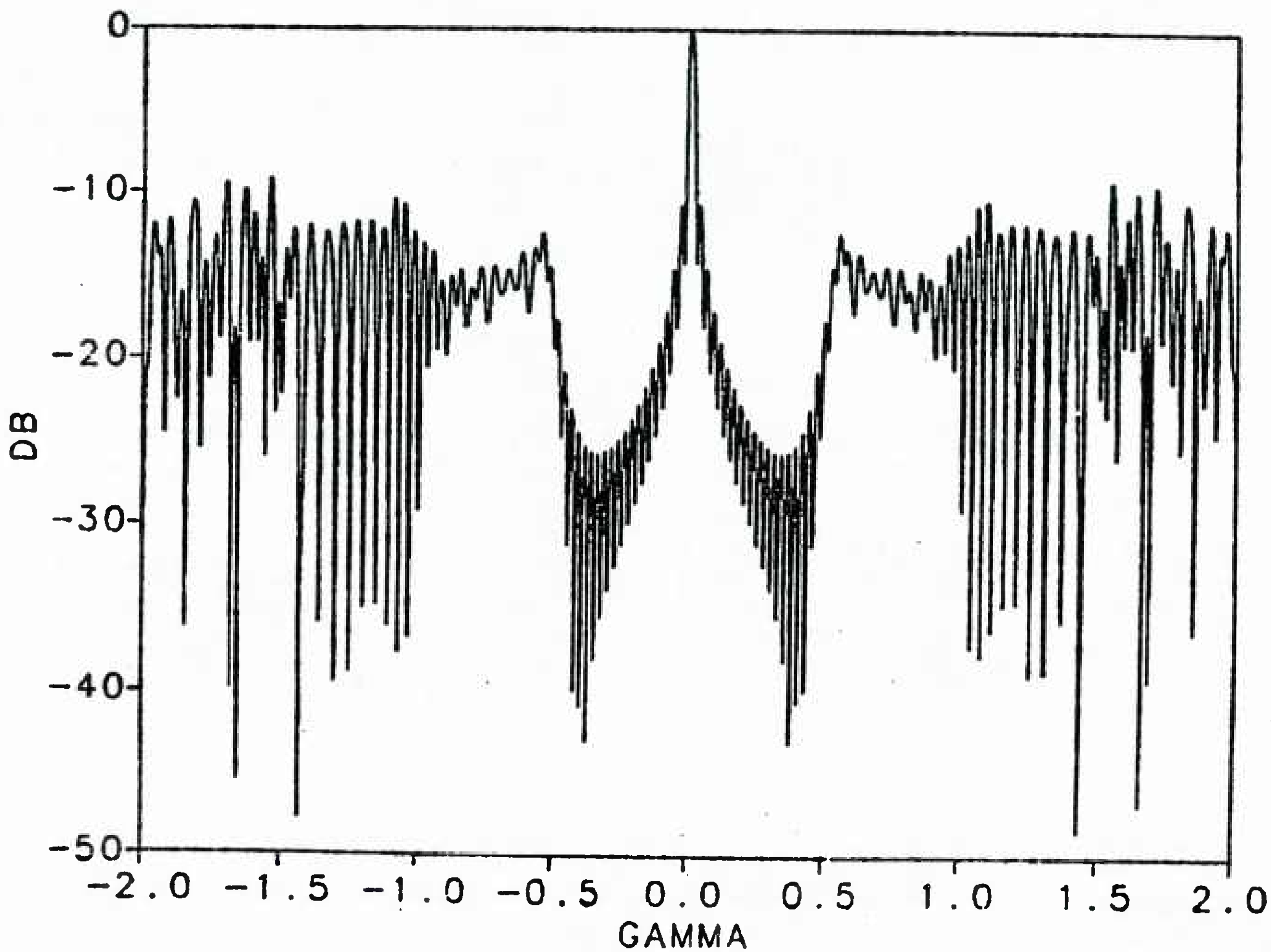

Figure 3. Universal Beam pattern of the Asymmetric Array

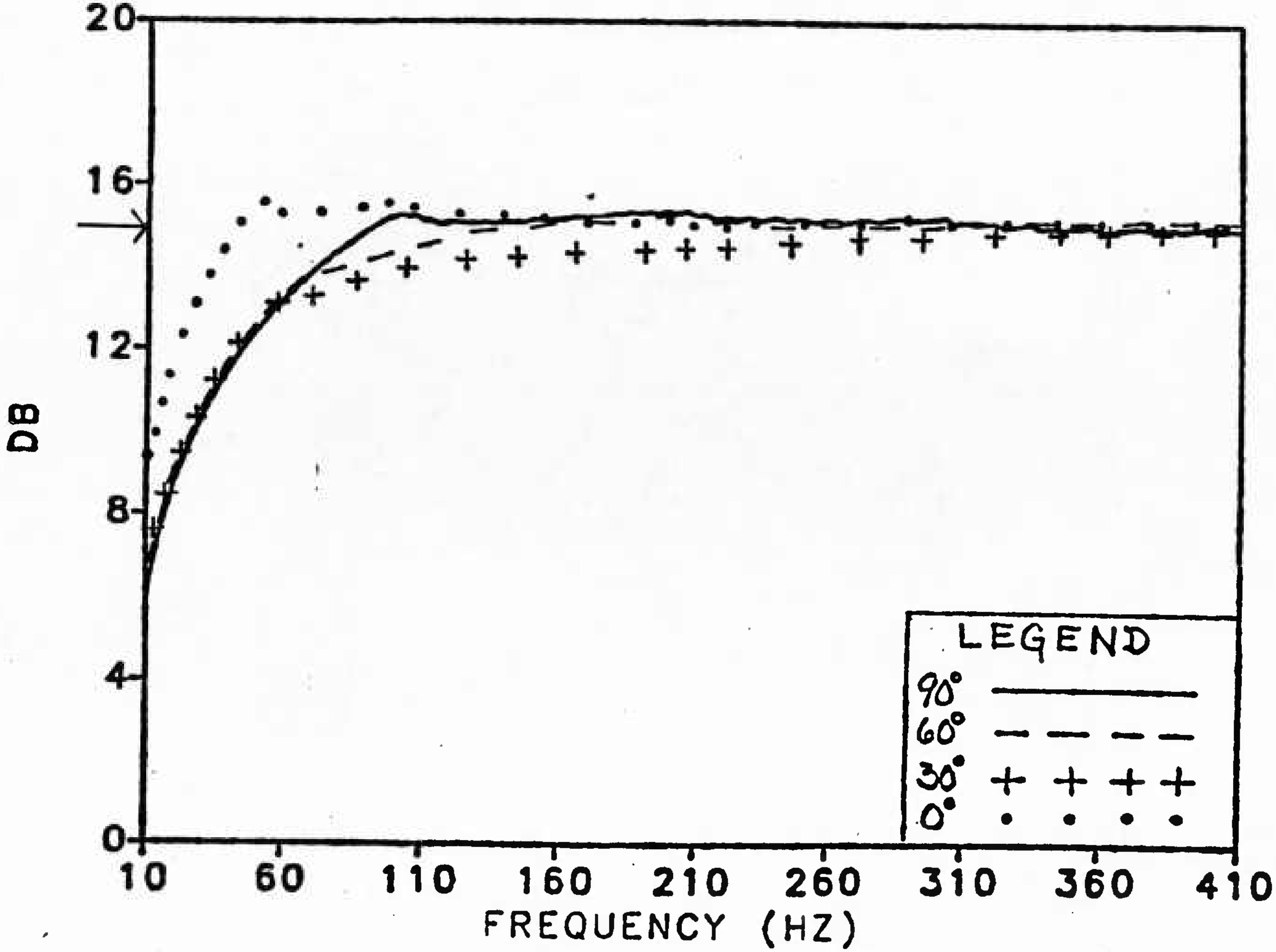

Figure 4. DI vs Frequency of the Asymmetric Array for Four Steering Angles

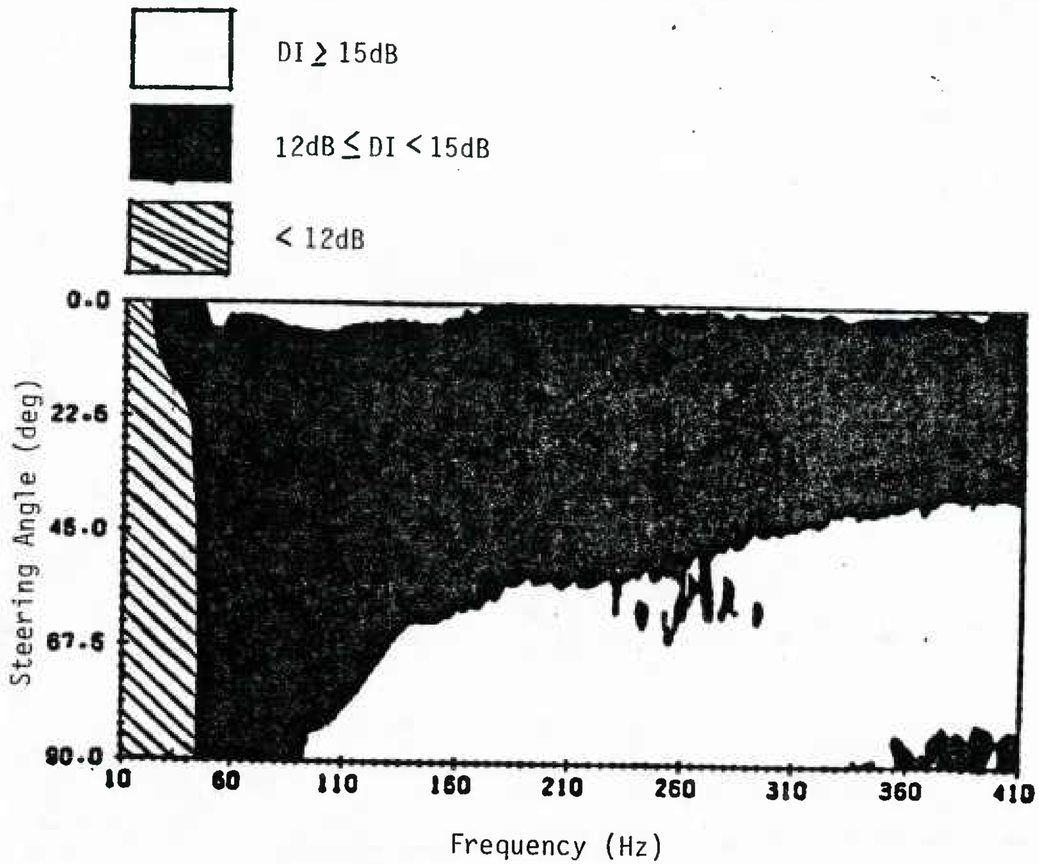

Figure 5. Contours of DI for the Asymmetric Array

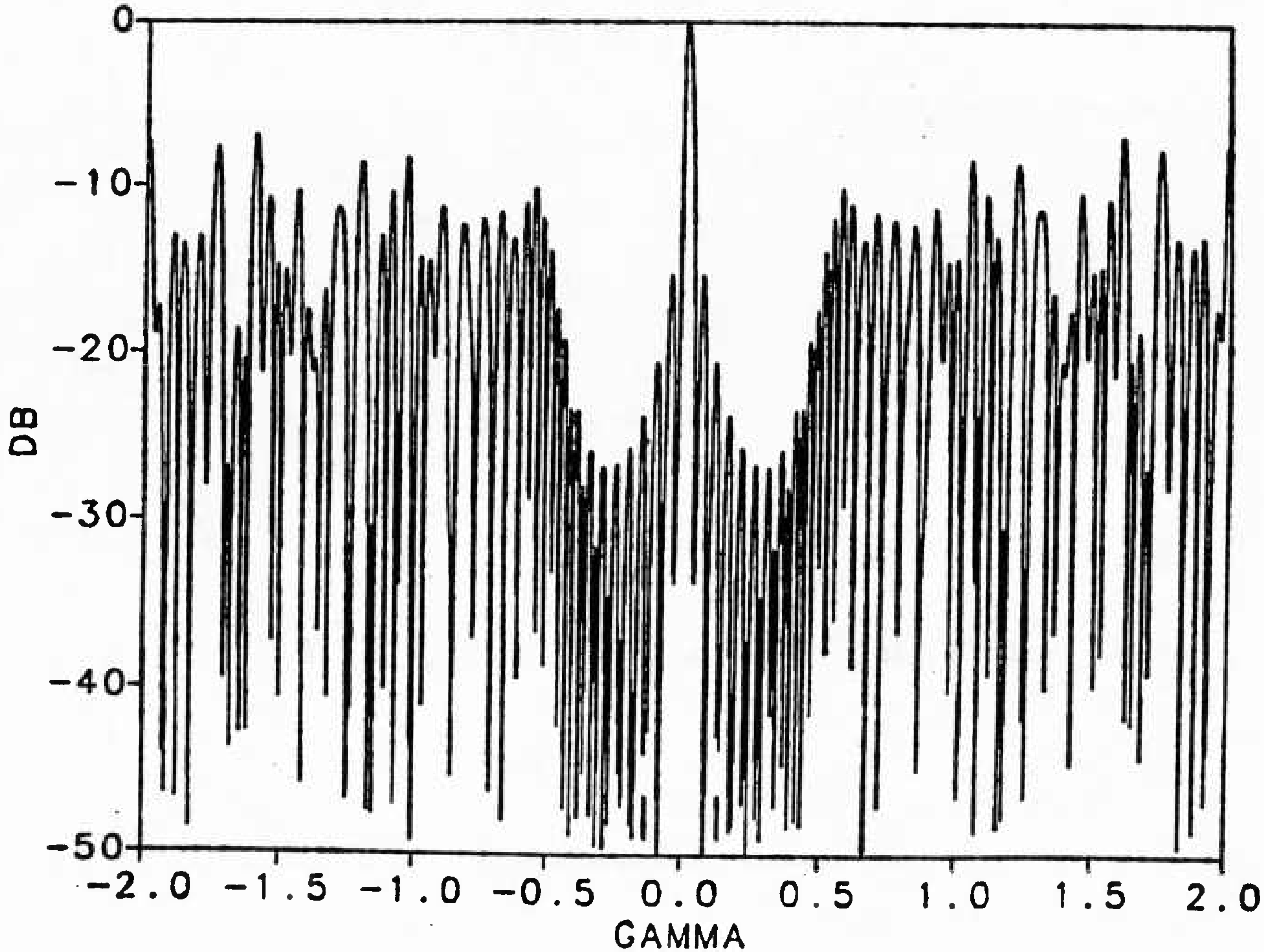

Figure 6. Universal Beam Pattern of the Symmetric Array

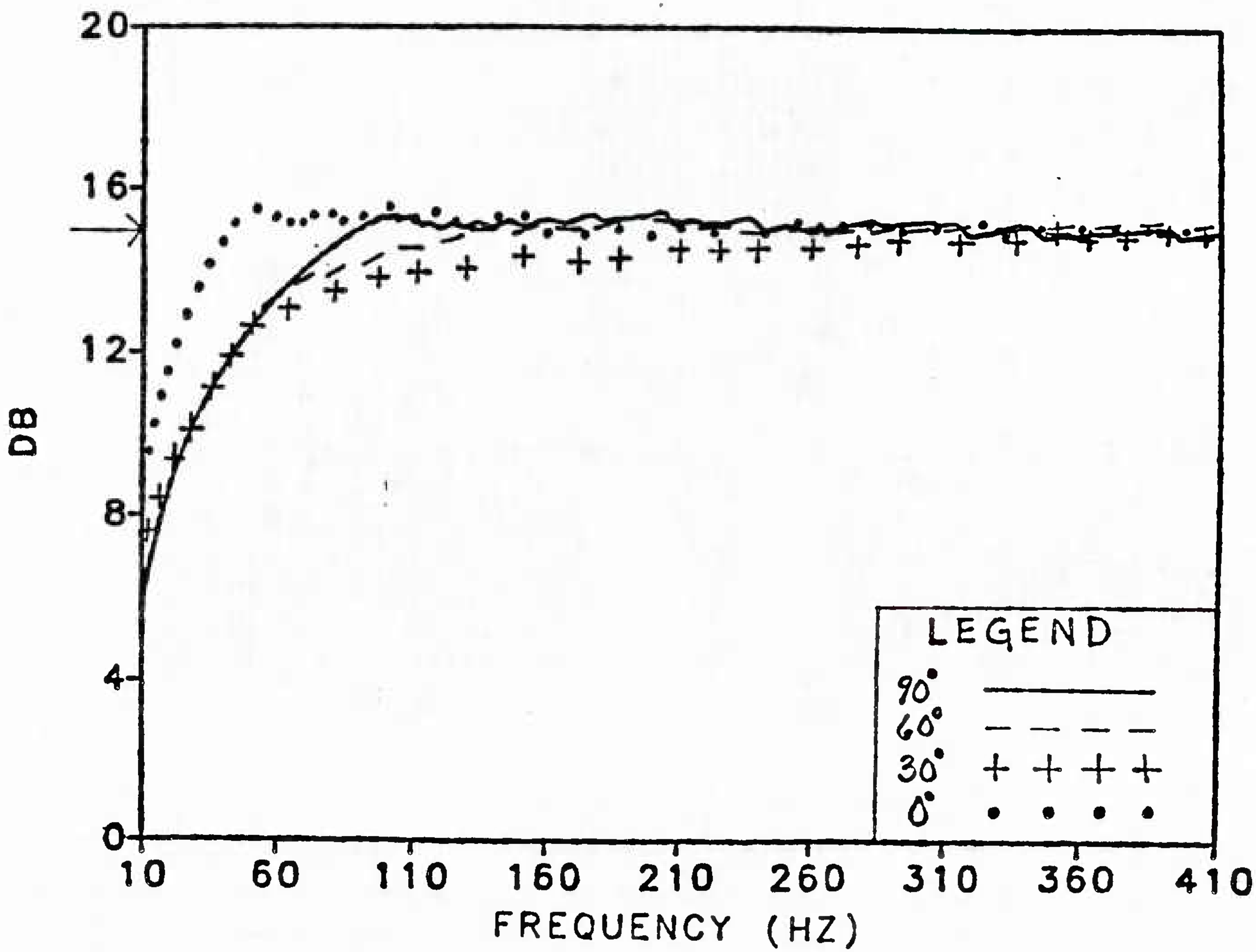

Figure 7. DI vs Frequency of the Symmetric Array for Four Steering Angles

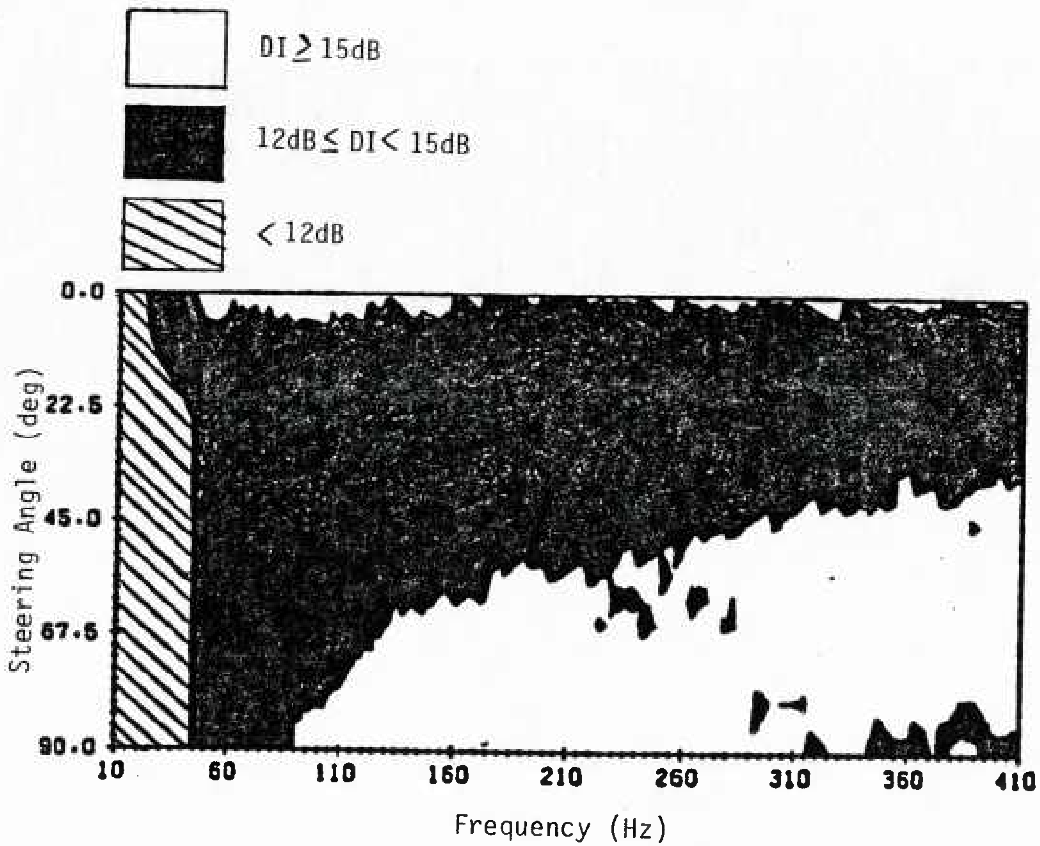

Figure 8. Contours of DI for the Symmetric Array

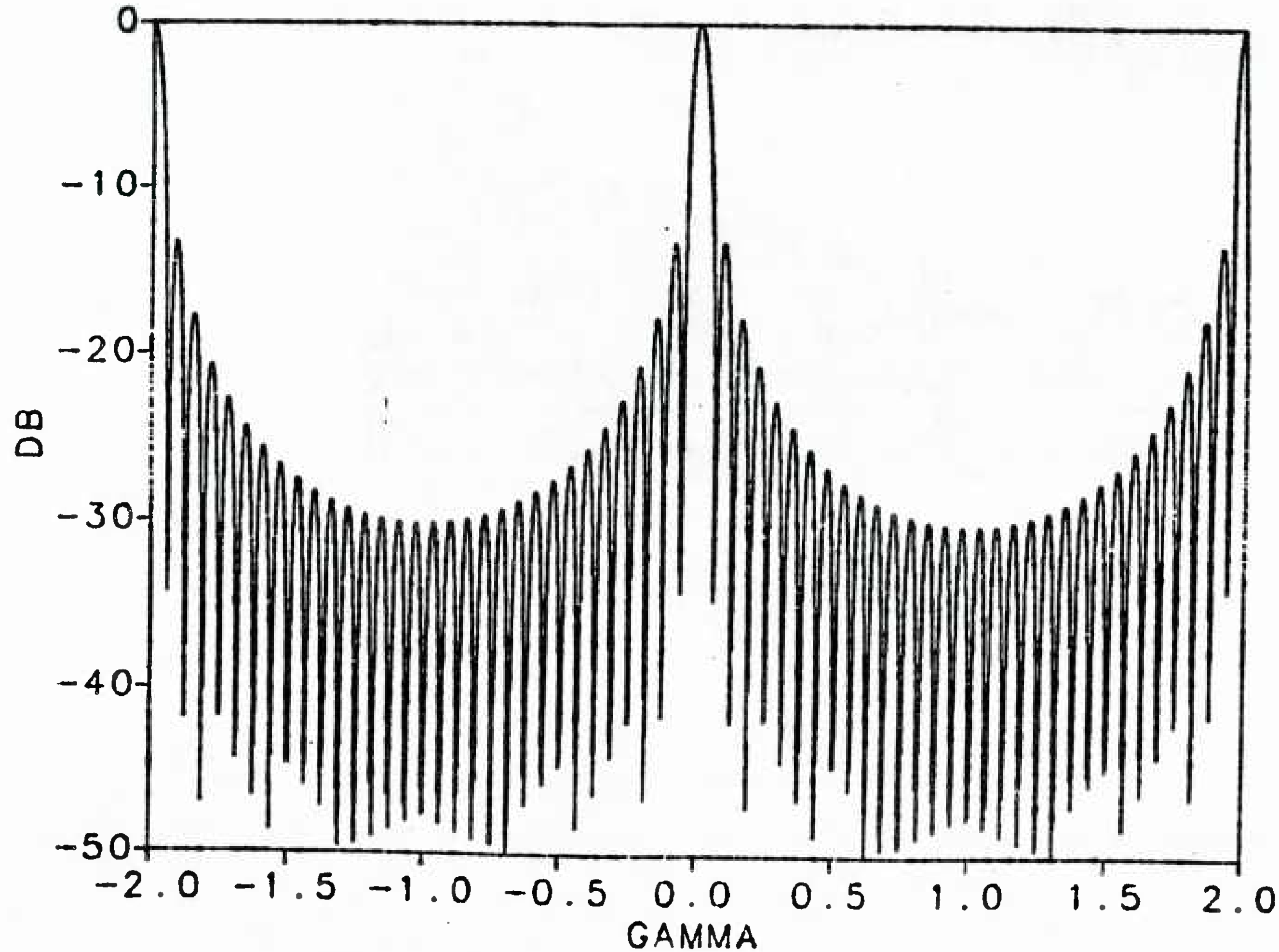

Figure 9. Universal Beam Pattern of the Equi-Spaced Array

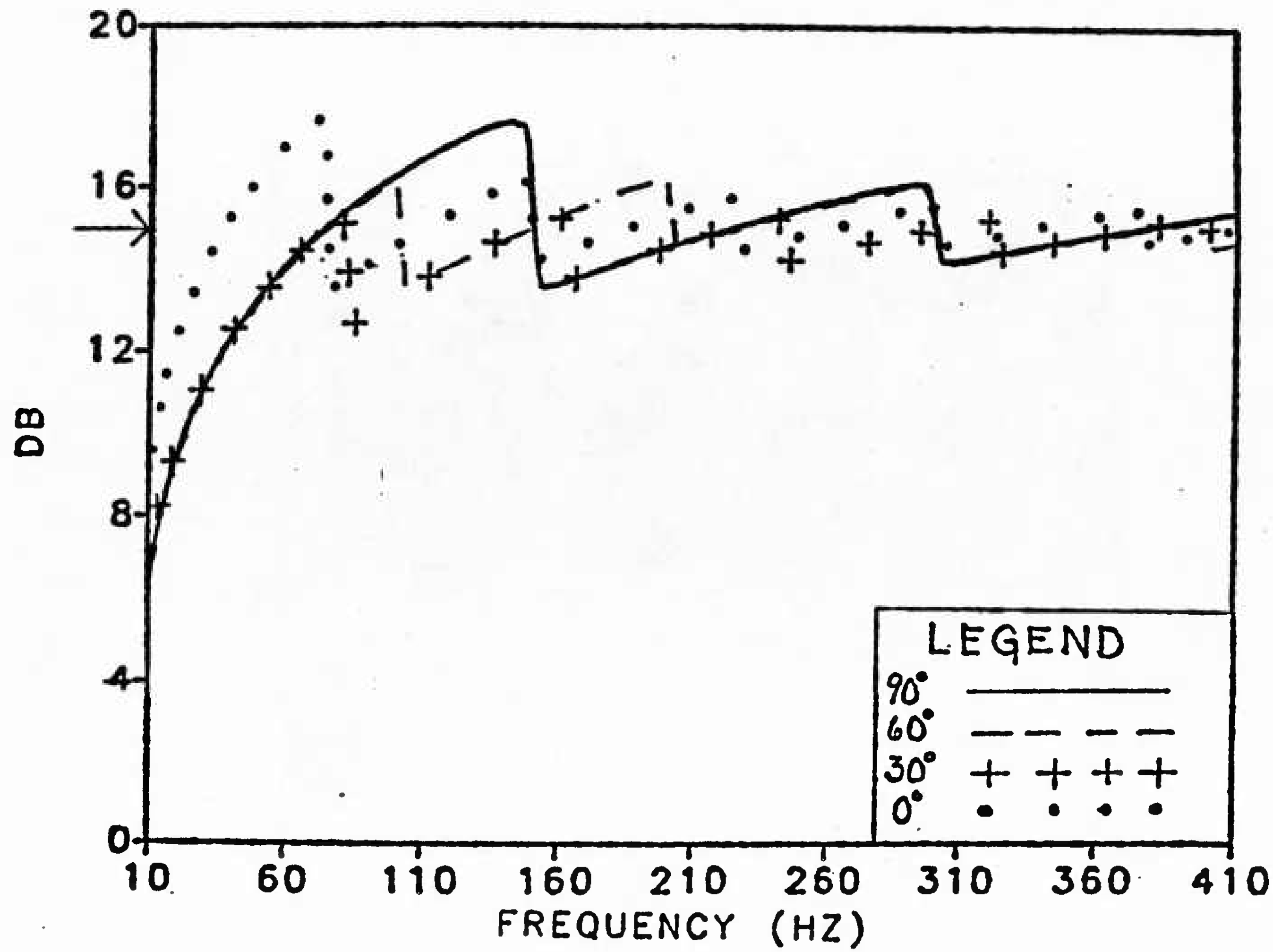

Figure 10. DI vs Frequency of the Equi-Spaced Array for Four Steering Angles

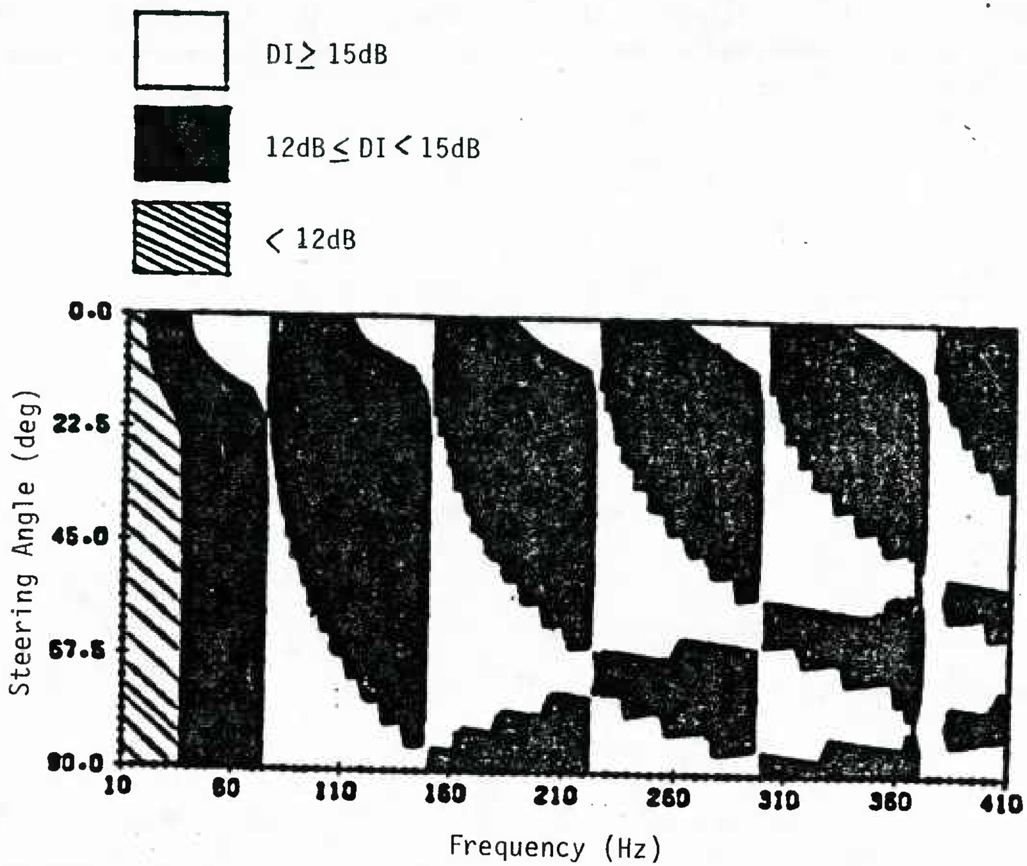

Figure 11. Contours of DI for the Equi-Spaced Array

REFERENCES

1. R. J. Urick, "Principles of Underwater Sound," (3rd edition), McGraw-Hill Book Co., NY, NY, 1983, pp. 42–43.
2. Applied Hydro-Acoustics Research, Inc., "Handbook of Array Design Technology, Vol. I," 30 June 76.

THE EFFECT OF HYDROPHONE CONFIGURATION ON
THE DIRECTIVITY INDEX OF A LINE ARRAY
Peter D. Herstein, 33A3
TM No. 841097
11 June 84
UNCLASSIFIED

External

DARPA	(CDR K. Evans)
CNO	(OP-095; -098; CAPT E. Young, CDR H. Dantzler, 952D) (4)
CNM	(SPO PM-2; MAT -0731 (T. Kooij), -0724 (CAPT J. Harlett), PM-4 (J. Sinsky)) (4)
NAVELEX	(R. Mitnick, J. Schuster, T. Higbee, 612) (3)
NAVSEASYSCOM	(SEA-63R; D. Porter, J. Shooter, 63R; P. Talbert, L. Kneesi, 63D) (5)
NAVPGSCOL	(1)
NORDA	(R. Lauer, 320; R. Martin, 150; D. Ramsdale, 345; G. Stanford, 115T, W. Worsley, 530; Library) (6)
NADC	(J. Howard, B. Steinberg, J. Gombali; M. Higgins, 3032; Library) (5)
NSWC	(R. Stevenson, M. Stripling, E. Hein, T. Harris; Library) (5)
NRL	(R. Dicus, R. Doolittle, 5160; Library) (3)
MPL	(F. Fisher, B. Williams) (2)
DTIC	(2)
DIA	(1)
ARL/UT	(N. Bedford, P. Vidmar) (2)
NOO	(W. Jobst, J. Allen, 7310; R. Merrifield, 7311; R. Swenson, 6200; R. Christensen) (5)
NOSC	(L. Arndt, D. Barbour, R. Bolam, C. Persons, 7133; R. Albrecht, J. McCarthy, 7134; M. Pederson, 724; R. Harris, 7231; Library) (9)
NAVAIR	(370B (E. Davis), 370J (W. Parigian)) (2)

External Copies - 55

THE EFFECT OF HYDROPHONE CONFIGURATION ON
THE DIRECTIVITY INDEX OF A LINE ARRAY
Peter D. Herstein, 33A3
TM No. 841097
11 June 84
UNCLASSIFIED

External

DARPA	(CDR K. Evans)
CNO	(OP-095; -098; CAPT E. Young, CDR H. Dantzler, 952D) (4)
CNM	(SPO PM-2; MAT -0731 (T. Kooij), -0724 (CAPT J. Harlett), PM-4 (J. Sinsky)) (4)
NAVELEX	(R. Mitnick, J. Schuster, T. Higbee, 612) (3)
NAVSEASYSCOM	(SEA-63R; D. Porter, J. Shooter, 63R; P. Talbert, L. Kneesi, 63D) (5)
NAVPGSCOL	(1)
NORDA	(R. Lauer, 320; R. Martin, 150; D. Ramsdale, 345; G. Stanford, 115T, W. Worsley, 530; Library) (6)
NADC	(J. Howard, B. Steinberg, J. Gombali; M. Higgins, 3032; Library) (5)
NSWC	(R. Stevenson, M. Stripling, E. Hein, T. Harris; Library) (5)
NRL	(R. Dicus, R. Doolittle, 5160; Library) (3)
MPL	(F. Fisher, B. Williams) (2)
DTIC	(2)
DIA	(1)
ARL/UT	(N. Bedford, P. Vidmar) (2)
NOO	(W. Jobst, J. Allen, 7310; R. Merrifield, 7311; R. Swenson, 6200; R. Christensen) (5)
NOSC	(L. Arndt, D. Barbour, R. Bolam, C. Persons, 7133; R. Albrecht, J. McCarthy, 7134; M. Pederson, 724; R. Harris, 7231; Library) (9)
NAVAIR	(370B (E. Davis), 370J (W. Parigian)) (2)

External Copies - 55